Chapter 14
Research on High Accuracy Prediction Model of Satellite Clock Bias

Xueqing Xu, Xiaogong Hu, Yonghong Zhou and Yezhi Song

Abstract Time basis of satellite navigation system is achieved by the satellite clock bias (SCB) prediction, while the SCB prediction accuracy will also affect the positioning accuracy of real-time navigation users. With the development of our Beidou satellite navigation system, the accuracy requirements of the SCB prediction is higher and higher, general quadratic polynomial extrapolation method have failed to meet the SCB forecast accuracy for each satellite. There is an urgent need to develop the SCB prediction program for each satellite, here we use a combined method of least squares and auto-regressive model (LS + AR) from the EOP forecasting, to predict and assess SCB with data from IGS. Results show that the combined LS + AR method can improve the SCB forecast accuracy effectively.

Keywords SCB prediction · Least squares · Auto-regressive model

14.1 Introduction

The long-term reliable forecasting of satellite clock error is an important prerequisite for autonomous navigation satellite orbit determination. Satellite clock cannot be compared to the ground time reference continuously while the satellites is in space orbit, when the satellite run into the arcs which cannot be observed by ground stations, the synchronization between satellite clock and the system time

X. Xu (✉) · X. Hu · Y. Zhou · Y. Song
Shanghai Astronomical Observatory, Chinese Academy of sciences,
Nandan Road No.80, Xuhui, Shanghai 200030, China
e-mail: xqxu@shao.ac.cn

X. Xu
Key Laboratory of Planetary Sciences, Chinese Academy of Sciences,
Shanghai 200030, China

J. Sun et al. (eds.), *China Satellite Navigation Conference (CSNC) 2014 Proceedings: Volume III*, Lecture Notes in Electrical Engineering 305,
DOI: 10.1007/978-3-642-54740-9_14,

can be only maintained by satellite clock himself, in order to get continuous satellite clock results, satellite clock error should be predicted necessarily [8, 5].

Prediction accuracy of the navigation satellites atomic clocks determines the ephemeris updated frequency, and the complexity and workload of satellite navigation ground operation control system, and the prediction accuracy is not only related with its physical characteristics, but also restricted by the complexity of the prediction algorithm [13, 15, 16]. Now the forecasting models about the satellite clock error at home and abroad are: (1) the linear model (LM) [9, 14] (2) quadratic polynomial model (QPM) [10, 19] (3) gray model (GM) [3, 4, 11, 17, Liu et al. 2006; Li et al. 2009] (4) Kalman filters model (Kalman Filter) [9, 14], and other methods etc. [6].

The above prediction methods are suitable for the deterministic part of the satellite clock error prediction (the part can be expressed by quadratic polynomial or periodic function), and ignoring the prediction of uncertainty part in satellite clock error (the part cannot be directly expressed by the model, also known as the residuals), resulting in prediction accuracy restriction in part of the satellite clock error series. For example, Cui (2005) pointed out that using the second-order polynomial model (QPM) for long-term forecasting of satellite clock error, the cumulative error will be extended with increasing forecast time, and cannot meet the large span clock error prediction. Lu et al. [12] pointed out that the application of gray system model GM (1, 1) to the prediction of satellite clock error, can reduce the satellite clock error epoch for modeling and improve forecast accuracy, which shows some advantages, but deficiencies in gray model will cause large errors in the actual forecasting sometimes. Xu (2009) gave the 90-day forecast results by quadratic polynomial method and gray model, the prediction accuracy is in the order of microseconds to 10 μs, while 1 μs clock error will lead to 300 m users range error (URE), which is difficult to meet the guidance accuracy needs during wartime.

This paper will learn the mature experience from forecasting on the Earth Orientation Parameters (EOP), use a jointed method by least-squares and auto regression model, abbreviated as LS + AR model, and predict the different satellite clock error sequences with IGS GPS satellite clock error data, to estimate the clock error accuracy of different prediction span. Specifically which is firstly analyze the data features of each satellite clock error sequence, and then create a specific model mainly contains the constant term, linear term, polynomial term and periodic terms, to isolate the main parts and residuals by fitting the clock error sequences, and the main parts and residuals parts are predicted by different methods, while the clock error value is derived by adding main term prediction and residuals prediction finally.

14.2 Methods

This paper selects AR (p) model but not the complex ARMA (p, q) as the forecasting model for clock error residuals series, mainly because the estimation accuracy of ARMA (p, q) model coefficient is poorer than the AR (p) model, and the total order (p + q) is higher the estimation accuracy is worse, while a higher-order AR (p) model is actually equivalent to a low-level ARMA (p, q) model, and the coefficient estimation of AR (p) model is more convenient and accurate compared with ARMA (p, q) model. We adopt the final prediction error criterion (FPE) to identify the order p of AR (p) model for each forecasting step, and use the Burg algorithm to solver the coefficients of AR (p) model, which means that the AR (p) model is updated for each forecasting step, and the new AR (p) model is more reasonable and accurate. The principles of fitting models and AR (p) model are briefly described as follows.

14.2.1 Fitting Model

Due to the different nature of main and residuals terms in the clock error sequence, the two parts usually need to be processed separately. We firstly identify the main periods of clock error sequence by the spectral analysis method, and then establish a model mainly contains the constant term, linear term, polynomial term and periodic terms, to isolate the main parts and residuals by fitting the clock error sequences, where the main periods are a week, 24 h and 12 h.

The fitting model is expressed as follows, which includes linear terms, polynomial and periodic terms:

$$Y_t = a + bt + ct^2 + \sum_{k=1}^{n} d_k \sin(2\pi t/P_k + \phi_k) + \varepsilon_t \tag{14.1}$$

where, a is the constant term, b is the linear coefficient, c is the quadratic coefficient, t is time, P_k, d_k and ϕ_k are period signals, amplitude and phase of periodic signals in satellite clock errors sequence, and ε_t is the noise.

According to the principle of least squares, the predictions of the clock error main terms can be extrapolated by the fitted model coefficients above, while the residual part in the clock error series is difficult to obtain by a simple linear model, and should be derived by other methods; here we select the classical autoregressive model (AR model).

14.2.2 AR Model

For a stationary random sequence $z_t(t = 1, 2, \ldots, N)$, the AR (p) model is expressed as follows,

$$z_t = \sum_{i=1}^{p} \varphi_i z_{t-i} + a_t. \tag{14.2}$$

where, a is zero-mean white noise, p is order of the model, $\varphi_1, \varphi_2, \ldots, \varphi_p$ are autoregressive coefficients, and obtained through solving the Yule-Walker equations by means of the Burg recursion [2] method.

The optimum order p is determined by Akaike's Final Prediction Error (FPE) criterion that corresponds to the smallest FPE [1],

$$\mathrm{FPE}(p) = \mathrm{P}_p(N + p + 1)/(N - p - 1), \tag{14.3}$$

$$\mathrm{P}_p = 1/(N - p) \sum_{t=p+1}^{N} \left(z_t - \sum_{j=1}^{p} \varphi_j z_{t-j} \right)^2. \tag{14.4}$$

14.3 An Example

Currently, GPS system totally contains 31 network working satellites, including 11 Block IIA satellites, 12 Block IIR satellites, seven Block IIR-M satellites and a Block IIF satellite. This example selects the PRN01, PRN05, PRN09, PRN16, PRN24 and PRN26 six satellites for analysis, which represent six types of GPS clock error: 1. Block IIA Cs; 2. Block IIA Rb; 3. Block IIR Rb; 4. Block IIR-M Rb; 5. Block IIF Cs; 6. Block IIF Rb.

This investigation employs the two weeks data set of clock error, during March 24, 2013 to April 6, 2013 from the International GNSS Service (IGS) official website for modeling and forecasting. The original data sampling interval is 30 s, in order to improve the computational efficiency; we adjust the data sampling interval into 6 min, and make the forecasting in the future 3 days containing short term to long-term changes in clock error. And the accuracy verification is obtained by comparing the model predicted values with clock error data, where the prediction spans are mainly selected as 2, 12, 24 and 72 h.

14.3.1 Prediction Error Estimates

We select the max absolute error (Max), mean absolute error (Mean), and root mean squared error (RMS) as the indicator to assess the prediction accuracy.

$$Max_i = \max\left|p_j{}^i - o_j{}^i\right|, \quad j = 1, \cdots, n. \tag{14.5}$$

$$Mean_i = \frac{1}{n}\sum_{j=1}^{n} \left|p_j{}^i - o_j{}^i\right| \tag{14.6}$$

Table 14.1 Statistics about the max, mean and RMS of GPS satellite clock bias prediction

PRN	T/h	Max/ns	Mean/ns	RMS/ns
01	2	0.190	0.062	0.076
	12	0.766	0.586	0.596
	24	2.538	2.333	2.336
	72	17.311	16.993	16.996
05	2	0.116	0.036	0.045
	12	0.744	0.276	0.331
	24	0.875	0.378	0.445
	72	6.092	6.539	6.097
09	2	0.844	0.317	0.452
	12	3.336	1.193	1.550
	24	4.109	1.451	1.805
	72	14.870	11.054	11.277
16	2	0.574	0.246	0.294
	12	0.860	0.436	0.515
	24	1.605	1.031	1.104
	72	8.006	7.154	7.165
24	2	2.983	1.172	1.452
	12	5.511	2.648	3.008
	24	10.635	6.293	7.001
	72	26.273	23.448	23.588
26	2	0.593	0.167	0.215
	12	1.546	0.880	0.906
	24	3.430	2.487	2.519
	72	16.902	14.765	14.790

$$RMS_i = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(p_j{}^i - o_j{}^i\right)^2} \tag{14.7}$$

where, o is the EOP observations, p is the EOP predictions, i is the prediction interval, n is the number of total predictions.

14.3.2 Result Analysis

For each clock error prediction interval (2, 12, 24 and 72 h), 100 predictions are made by means of LS + AR method, and the maximum absolute error (Max), mean absolute error (Mean) and the mean square error (RMS) are calculated and listed in Table 14.1, the unit is nanoseconds (ns).

As an example, Figs. 14.1 and 14.2 displays the clock error original sequence, decomposed fitting sequence and residuals of typically rubidium clock error

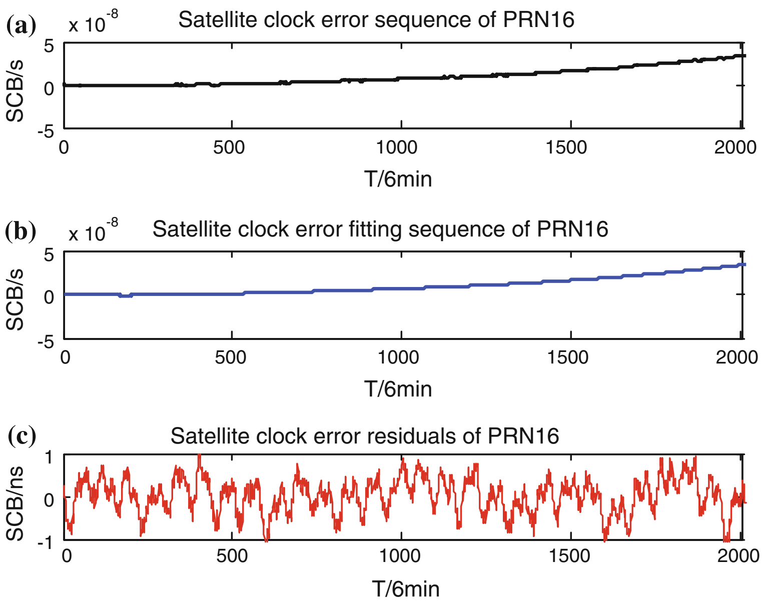

Fig. 14.1 SCB series, model fitting series and residual series of PRN16

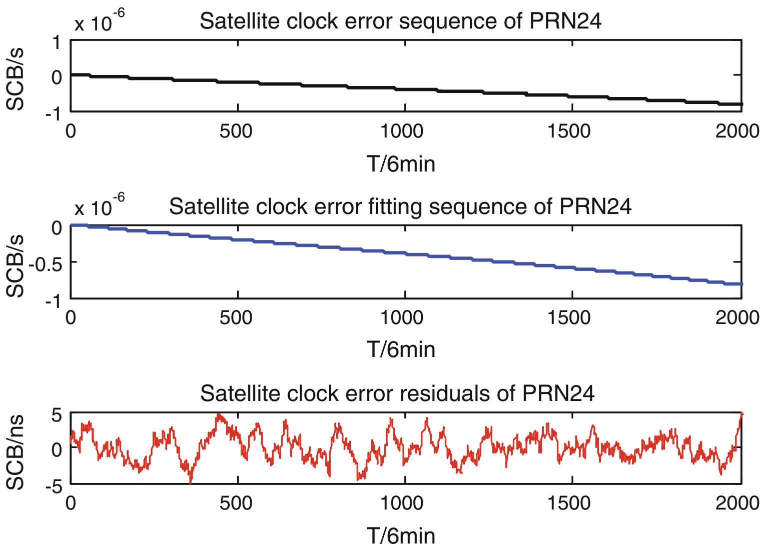

Fig. 14.2 SCB series, model fitting series and residual series of PRN24

sequence from PRN16 satellite and cesium clock error sequence from PRN24 satellite.while Figs. 14.3 and 14.4 shows the 24 h prediction result of the two satellite clock error series.

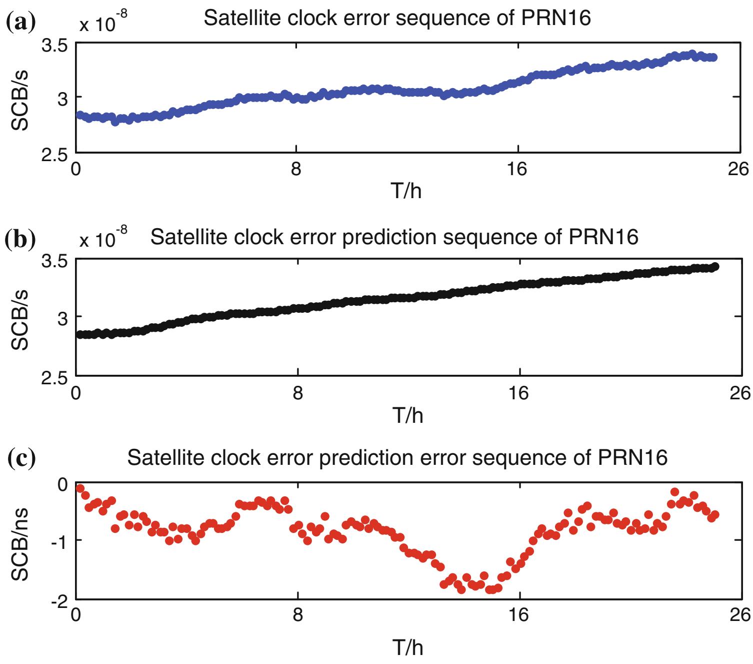

Fig. 14.3 24 h SCB series, prediction series and prediction error of PRN16

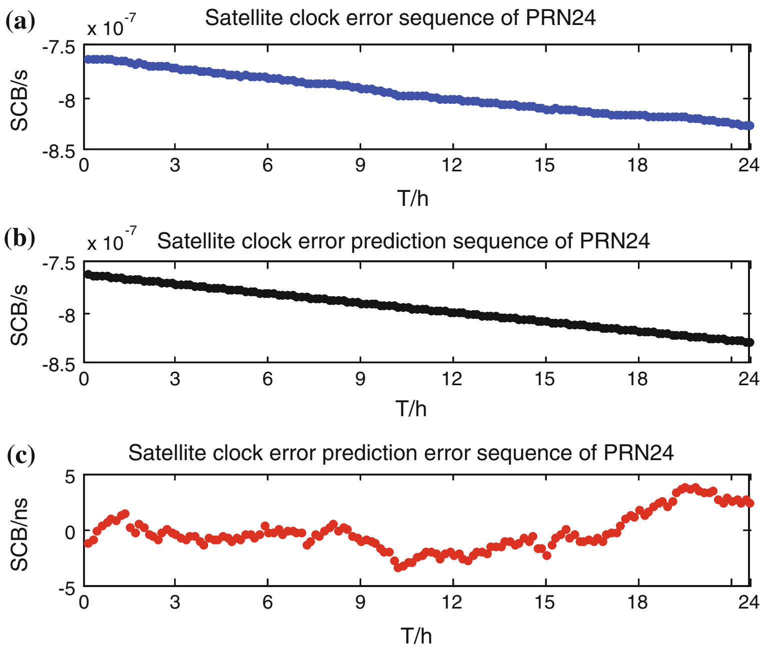

Fig. 14.4 24 h SCB series, prediction series and prediction error of PRN24

We can get some information from Figs. 14.1 and 14.2, that the PRN16 rubidium clock error sequence shows more stable residuals after model fitting, which is in the order of 2 ns; while PRN24 cesium clock error sequence remain a bigger residuals after model fitting about 10 ns, which shows that the stability of cesium clocks is worse than the rubidium clock. Meanwhile, it is shown from Table 14.1 and Figs. 14.3, 14.4, that PRN09, PRN24 are cesium clock, whose 12 h short-term forecast accuracy is poor, in the order of nanoseconds; while PRN01, PRN05, PRN16, PRN26 are rubidium clocks, the 12 h forecast accuracy of them are Higher, and are both in the sub-nanosecond magnitude. And for 24 and 72 h long-term forecasting, the cesium clock PRN24 perform most unstable, the RMS are 7 and 23.5 ns respectively, while the rubidium clock PRN05 and PRN16 perform best, which remains a nanosecond magnitude for the medium and long-term forecast accuracy.

14.4 Conclusion

In this study, we perform the short and medium-term predictions of the GPS clock error by means of LS + AR method, based on the IGS clock error sequence during March 24, 2013 to April 6, 2013. The prediction intervals range from 1 to 72 h. We get two information from the GPS satellite clock error sequence, on the one hand, the different types of satellite clock showing a law that is, the stability of rubidium clock is higher than cesium clock overall, and rubidium clock shows higher prediction accuracy than cesium clock for short-term and long-term forecasting. On the other hand, the LS + AR model reduce error accumulation by modeling each data series differently, which can effectively improve the prediction accuracy of the satellite clock error.

Acknowledgments The research is supported by the NSFC grant (11303073,11373057, 11073045), STCSM (06DZ22101). We thank the International GNSS Service (IGS) for providing the GPS satellite clock error data.

References

1. Akaike H (1971) Autoregressive model fitting for control. Ann Inst Stat Math 23:163–180
2. Brockwell PJ, Davis RA (1996) Introduction to time series and forecasting. Springer, New York
3. Cui X, Jiao W (2005) Grey system model for the satellite clock error predicting. Geomatics Inform Sci Wuhan Univ 30(5):447–450
4. Dai W, Jiao W, Li W (2009) Research on prediction of GPS block IIR atomic clock error. J Geodesy Geodyn 29(4):111–115
5. Guo H (2004) Frequency analysis theory and methods of navigation satellite atomic clocks. Information Engineering University, Zhengzhou

6. Guo H, Yang S, Yang Y (2007) Numerical prediction methods for clock difference based on two-way satellite time and frequency transfer data. Geomatics Inform Sci Wuhan Univ 32(1):43–46
7. Wei L, Chen P, Jinzhong M (2009) Application of gray system model in short-term predicting for the satellite clock error. J Surveying Mapp 6:32–35
8. Liu L (2004) Relativity theory of time comparison and high accuracy time synchronization technology. Information Engineering University, Zhengzhou
9. Liu X, Wu X, Zhao R et al (2011) Analysis of long-term clock bias forecast effects of several time prediction models. J Surveying Mapp 1:15–18
10. Liu X, Wu X, Tian Y (2010) Study on atomic prediction of time based on interpolation model with tchebytchev polynomials. J Geodesy Geodyn 30(1):77–82
11. Liu Y, Dang Y, Zheng Z (2011) Study and improvement for the model in satellite clock error long-term forecast. Hydrographical Surveying Charting 31(2):21–23
12. Lu X, Jia X, Cui X (2006) The optimization method of gray system theory for the satellite clock error short-term predicting. Eng Surveying Mapp 15(6):12–14
13. Wang Q, Li L, Gong Y (2010) Study of GPS satellite clock's behaviours and prediction. Sci Survey Mapp 35(2):36–39
14. Xu J, Zeng A (2009) Application of ARIMA (0, 2, q) model to prediction of satellite clock error. J Geodesy Geodyn 29(5):116–120
15. Zheng Z, Lu X (2008) Comparison and precision analysis of several GPS satellite clock bias prediction methods. J Shandong Univ Sci Technol 27(4):6–15
16. Zhu L (2006) Research on time comparison and performance analysis of atomic clocks. Information Engineering University, Zhengzhou
17. Zhu L, Wu X, Li C (2007) The two models for prediction of satellite clock error analysis. J Spacecraft TT&C Technol 26(3):39–43
18. Zhu X, Xiao H, Yong S et al (2008) The Kalman algorithm used for satellite clock offset prediction and its performance analysis. J Astronaut 29(3):966–970
19. Zhu L, Li C, Liu L (2009) Research on methods for prediction clock error based on domestic hydrogen atomic clock. J Geodesy Geodyn 29(1):148–151